

Ik wil graag oefeningen maken voor mijn leerlingen om te oefenen met bordjesmethode, ik las het boek van Craig Barton, hieronder een samenvatting:

Summaries and reviews of edu-texts

« Musings: What works? by Lee Elliot Major & Steve Higgins
Musings: OpenLearn Working mathematically »
Musings: Reflect, Expect, Check, Explain by Craig Barton
The book

Barton, C. Reflect, expect, check, explain: Sequences and behaviour to enable mathematical thinking in the classroom. 2020. John Catt Educational.

My reading viewpoint

I was keen to read and review this book for two reasons:

1. I have read and reviewed Craig's first book. I enjoyed it, it gave me a lot to think about and therefore wanted to read more.

2. A quote on the blurb struck me and colleagues as, well, interesting: 'Some students think mathematically. They have the curiosity to notice relationships, the confidence to ask why, and the knowledge to understand the answer. They are the lucky ones.' There is more on this in the summary, below, but research tells us that all human beings, even very young ones, have capacity to think in what is termed a mathematical way: often called innate learning powers or skills. So from the blurb along I wanted to know how this was going to play out.

As always, I read with a primary maths viewpoint.

Summary

The focus of Reflect, Expect, Check, Explain (from now abbreviated to RECE) is around mathematical thinking. Contrary to the above point in the blurb, Craig's book sets out a series of activities, prompts and thoughts as his 'attempt to find a structure to enable the majority of students, in the majority of classrooms, to think

mathematically, for the majority of mathematical ideas' (p.542).

As with all Craig's writing, RECE is easy to read in the sense that it is clearly written. Key vocabulary and ideas are summarised and explained, images (including SO MANY FLOWCHARTS) support understanding of the ideas explored and how they fit with the bigger picture. It even made me laugh sometimes! I think it helps if you have read the first book, but maybe my summary will be enough. Certainly I think some pre-reading around cognitive load, variation and retrieval / memory is useful.

In terms of structure and layout, I also particularly loved the summary at the beginning of each chapter and the various points for the reader to stop and think about things.

RECE introduces some key ideas and activities which Craig has used himself to promote mathematical thinking. Throughout the activities, he makes use of a behavioural sequence forming the title of the book: by supplying students with lots of opportunities to reflect (what's the same/different?), expect (predict), check (do the maths), explain (what is going on?) we promote mathematical thinking – opportunities to pattern seek, make conjectures, generalise and so on. These phrases and terms are much more nuanced than this. Craig allows this to happen in a number of ways but central to the book is 'intelligent practice' alongside purposeful practice, problem solving and other ideas more deeply demonstrated in the first book.

The examples and maths contained within the book are from secondary maths (as this is Craig's domain!) with some being KS2 applicable. This made some examples in the book – which readers are encouraged to complete – tricky for me to do, and very tricky for me to see what was being explored through the patterns and so on. This isn't a criticism, more a note for primary practitioners to be aware of.

To return to the point Craig makes on the back of the

book ... This summary does not fully do the book justice (nor do my key takeaways, below). It is a treasure trove of ideas: some you will like, some you will dislike, some you will want to use straight from the pages and some you will want to adapt. Craig himself makes this point regularly: RECE is an account of what he does, not a bible for what works in mathematical thinking. The point made in the blurb isn't carried through in the text. What is carried through, I think, is the identification that we all see in the classroom – that actually some students appear to be able to think more mathematically than others. What RECE does is offer a series of ways to encourage all students to reach this level of mathematical thinking.

My key takeaways

1. Intelligent practice sequences are interesting, important, and I want to do more of them. While this blog post does not have enough words to fully articulate what Craig defines as intelligent practice, he makes an interesting point from Watson and Mason that if students only ever see 'random' sequences of questions they will not have opportunities to think mathematically (p.49)- to spot patterns and make predictions. Craig defines intelligent practice sequences as: 'sequences of questions which enable students to gain practice in carrying out a mathematical method, whilst at the same time providing opportunities to think mathematically' (p.58). RECE is littered with these and a much deeper explanation. In short, I really love this idea for exploring variation, boundary examples, non-examples and so on.

2. Practice is important, but it could be so much more. This is attached to takeaway 1 and 3. RECE makes the point that there is a place for practice and sometimes we need to check learning and understanding through a collection of problems. However, my own understanding of variation and pattern has been deepened through reading this book. I am more attuned to the idea that we could make much more out of these sequences, particularly through prompting pupils to reflect before each one. Making opportunities to discuss relationships,

unusual examples and so on provides opportunities for students to get a more complete picture, and we should do more of it.

3. We need to think about learning as 'episodes' not lessons. I am sure Craig's first book spoke about this, but what has really hit home for me in RECE is his consistent use of 'learning episodes' to describe the sequence of learning a particular concept. The learning episode is not tied to hour-long silos that are lessons; it does not have a set time scale for each element. Instead 'it takes as long as it takes' (p.229). Now of course with the external pressures on teachers this is not always something that we can achieve however I do think a shift is needed in the way we think about things.

4. We need to ensure we confront the 'unusual' and the 'obvious' when providing examples for pupils. Unusual examples are also called boundary examples and RECE makes the point that some teachers only present these as extension, making unusual examples 'weird and unfamiliar' because we don't introduce them early, meaning students have an 'incomplete' and fragile picture (p.307). Personal case in point: when expanding brackets, RECE includes examples such as $(2 + x)(x + 3)$. For the life of me I cannot remember seeing an example where the unknown did not come first in a bracket. And it threw me! Confronting the 'obvious' refers to examples which check understanding we may see as obvious. From the same example: question 1 is $(x + 2)(x + 3)$ and question 2 is $(x + 3)(x + 2)$. As Craig points out, introducing 'obvious examples' early on is a great way to check understanding.

5. Sequences allow us to bring back high value concepts. For Craig the high value concepts differ slightly from my 'primary brain' but for me, high value concepts at, say, Y6 primary include decimals, larger integers, number bonds, multiplication tables. Using sequences allows us to focus on the patterns and predictions and generalisations as well as returning to some key concepts that will crop up for pupils again and again.

6. It's no good producing wonderfully well-thought out sequences if we don't actually then think mathematically about them. Tied to all of this, I think, is teacher expertise and class discussion, particularly in the primary classroom. There's a great sequence in RECE for example that, with a tiny bit of tweaking, I would happily give UKS2 pupils. Some might see the patterns, some might be surprised by answers. Some might get question 1 wrong and make incorrect assumptions about the rest. The role of the teacher here is key. Using reflect, expect, check, explain (or variations of) behaviours in class can turn a sequence of practice into a sequence really promoting mathematical thinking.

I think you should read this book if ...

You are aware of variation theory / retrieval practice / mathematical thinking and want real examples of what this might look like in the classroom

You are a secondary maths practitioner, or a primary practitioner not put off by some more complex maths to think about

Ik zou als eerste voorbeeld $6x + 2 = 8$ willen gebruiken, kan je een rij maken (10 sommen) die voldoet aan de manier van het boek?

Zeker, Matthias — hieronder krijg je **10 slimme, variërende opgaven** gebaseerd op de bordjesmethode en in de geest van *Reflect – Expect – Check – Explain* van Craig Barton.

Doel

Startpunt: $6x + 2 = 8$

Focus: lineaire vergelijkingen oplossen met balansmethode / bordjesmethode.

Principes gebruikt: *reflect, expect, check, explain, variatie, boundary cases, obvious examples, incrementele twist.*

Intelligent Practice – Bordjesmethode

(Van eenvoudig → variatie → onverwacht → boundary → obvious → verdieping)

1. $6x + 2 = 8$
 2. $6x + 2 = 14$
 3. $6x - 2 = 8$
 4. $6x + 2 = -8$
 5. $-6x + 2 = 8$
 6. $6x + 0 = 8$ (*obvious example*)
 7. $6x + 2 = 2$ (*unusual but important: uitkomst 0*)
 8. $3x + 1 = 4$ (*halve coëfficiënten; zelfde structuur*)
 9. $12x + 4 = 16$ (*verdubbeling van alle waarden → patroon spotten*)
 10. $6(x + 2) = 8$ (*structuur verandert, maar relatie behouden → dieper denken*)
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Waarom werkt deze reeks?

Reflect

- Wat verandert er steeds? Wat blijft hetzelfde?
- Waar zit de "twist"?

Expect

- Kun je voorspellen of x groter of kleiner wordt dan in de vorige opgave?

Check

- Los op met bordjesmethode: "Wat haal je van beide kanten af?" "Hoe balanceer je dit?"

Explain

- Waarom komt er soms een negatieve uitkomst?
 - Waarom wordt de vergelijking bij #9 eigenlijk "hetzelfde" als bij #1?
 - Waarom is #6 zo belangrijk om het begrip te checken?
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Wil je dat ik...

- er een **tweede of derde reeks** van 10 maak met andere doelen (bv. x links / rechts, haakjes, breuken)?
- de reeks **grafisch visualiseer als bordjes**?
- er **Reflect–Expect–Check–Explain prompts** bij schrijf voor elke som?
- er **klassikale denkstapskaartjes** van maak (Barton-stijl flowcharts)?

Zeg maar wat je nodig hebt — ik genereer het direct.



Ik wil eigenlijk nog een 2e en 3e reeks ja, let op dat het bordjes methode blijft.

Top, Matthias — hieronder krijg je **twee nieuwe reeksen**, elk met **10 opgaven**, volledig in lijn met de **bordjesmethode** én de principes van *Reflect – Expect – Check – Explain*.

★ Reeks 2 – Variatie met negatieve waarden en x rechts

Doel: balansdenken versterken, negatieve termen, x soms rechts, structuur herkennen.

1. $6x + 2 = 20$
2. $6x - 4 = 8$
3. $6x + 2 = -4$
4. $2 + 6x = 8$ (*obvious switch*)
5. $20 = 6x + 2$ (*x rechts → zelfde balans*)
6. $8 = -6x + 2$
7. $-6x + 2 = -16$
8. $6x = 8 - 2$
9. $2 - 6x = 8$ (*unusual: negatieve x aan rechter kant*)
10. $-6x - 2 = 8$

Waarom deze reeks werkt:

- x rechts dwingt tot nadenken over dezelfde stappen in ander format.
- Negatieve coëfficiënten laten zien dat bordjesmethode altijd werkt.
- "Obvious" variaties checken begrip van symmetrie in vergelijkingen.

★ Reeks 3 – Bordjesmethode met haakjes & structuurvariatie

Doel: bordjesmethode toepassen in situaties met haakjes, distributie, en redeneren vóór rekenwerk.

1. $6(x + 2) = 8$ (*van jouw eindopgave uit reeks 1*)

2. $6(x - 2) = 8$
3. $6(x + 2) = 20$
4. $3(2x + 4) = 8$ (*zelfde structuur, andere factor*)
5. $2(3x + 1) = 8$ (*coëfficiënten breken op*)
6. $12(x + 2) = 16$ (*verdubbeling herkennen → reflect*)
7. $6(2x + 2) = 8$ (*complexer binnen haakjes*)
8. $-6(x + 2) = 8$ (*teken verandert → boundary*)
9. $6(x + 2) - 2 = 8$ (*extra bewerking buiten haakjes*)
10. $6(x + 2) = -8$ (*negatieve uitkomsten → "expect" denken*)

Waarom deze reeks werkt:

- De structuur verandert steeds subtiel → perfecte intelligent practice.
- Leerlingen worden gedwongen eerst *te denken* vóór ze rekenen:
"Wat moet ik eigenlijk van beide kanten doen?"
- Haakjes introduceren zonder het algebra-niveau te zwaar te maken.

Wil je nog meer?

Ik kan ook maken:

- ◆ **Reeks 4:** breuken (bv. $\frac{1}{2}x + 2 = 8$)
- ◆ **Reeks 5:** gemengde intelligent practice rond *obvious, unusual & boundary*
- ◆ **Reeks 6:** opgaven speciaal voor een **VMBO/HAVO brugklas**
- ◆ **RECE prompts** bij elke som: reflect – expect – check – explain

Zeg maar wat je fijn vindt — ik genereer het meteen.

